Chained Function Filters

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***Abstract*— In this letter we describe a new family of filter transfer functions called *Chained Function* (CF) filters that have the property of having a reduced sensitivity to manufacturing errors. The reduction is achieved by controlling the position of the return loss zeros while maintaining a given maximum ripple in the inband response. The formulation is given in detail together with an example of application confirming the achieved reduction. The CF family of filters can be used effectively to extend the range of applicability of state-of-the-art tuningless implementations toward higher frequencies or narrower bandwidths.**

***Index Terms*—Microwave filters, network theory, reduced sensitivity.**

I. Introduction

MOST OF THE microwave filters which are currently manufactured for both ground and space applications are of the Chebyshev family. It is in fact well known that this type of filter produces the best out-of-band rejection for an equiripple inband response with a given filter order. In practical applications, however, the inband response does not need to be equiripple so that the parameter that is specified is the maximum inband insertion loss variation. It is also well known that for this family of filters, a critical factor in the achievement of a "one-pass" tuningless manufacture is the relative separation in frequency of the return loss zeroes which are distributed within the pass-band of the filter in such a way as to provide the equiripple response.

In this letter we describe a new family of filters, the Chained Function Filters (patent pending), which exhibits a reduced sensitivity to manufacturing errors while maintaining a maximum inband insertion loss ripple.

II. The Chained Function Formulation

The approximation of rational functions representing filters is well known (see [1] for instance). A convenient representation is in the form

$$S_{21}(p)^2 = \frac{1}{1 + h^2 G_m^2(p)} \tag{1}$$

where the function $G_m(p)$, the generating function, is a polynomial of degree n.

For Chained Function filters we define a new class of generating functions where $G_m(p)$ is given by the product of functions, called seed functions, obtaining

$$G_m(p) = \prod_{i=1}^{k} C_{n_i}(p) \tag{2}$$

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and where the total order of the filter is given by the sum of the degrees of the constituent seed functions.

The new transfer function thus obtained exhibits a decreased sensitivity to variations of the location of the poles. This is due to the increased multiplicity that results in an additional zero in the numerator of the derivative of (1).

To continue, consider next the seed function $C(p)$ as being a Generalized Chebyshev function. It is well known that this function can be defined as

$$C_{n_i}(p) = \cos\left(\sum_{l=1}^{n_i} \cos^{-1} \frac{p + \frac{1}{p_l}}{i\left(1 - \frac{p}{p_l}\right)}\right) \tag{3}$$

where pl are the transmission zeroes of the filter. This may be manipulated into a rational function form, using known techniques, to give

$$C_{n_i}(p) = \frac{N_{n_i}(p)}{D_{l_i}(p)} \tag{4}$$

with

$$N_{n_i}(p) = \prod_{l=1}^{n_i} (p - z_i) \tag{5}$$

and

$$D_{l_i}(p) = \prod_{o=1}^{l_i} (p - p_o). \tag{6}$$

Taking m such functions, we can then define the overall filtering function by evaluating the product of the individual seed functions. Realizability is ensured by the fact that each function in the product is realizable. Having derived the chained functions as a generating function, it remains only to derive the global transfer function in the rational form, namely

$$S_{21}(p)^2 = \frac{D_l^{2m}(p)}{D_l^{2m} + h^2 N_n^{2m}(p)}. \tag{7}$$

We then select only the left half-plane poles of the transfer function and reconstruct to a realizable form for synthesizing the filter. The poles occur in the usual alternating left–right half-plane configuration, and, for the symmetrical case for instance, we may equate the polynomial form of the transfer function with the even- and odd-mode admittance form from Bartlett's Bisection Theorem:

$$S_{21}(p) = \frac{Y_e - Y_o}{(1 + Y_e)(1 + Y_o)} \tag{8}$$

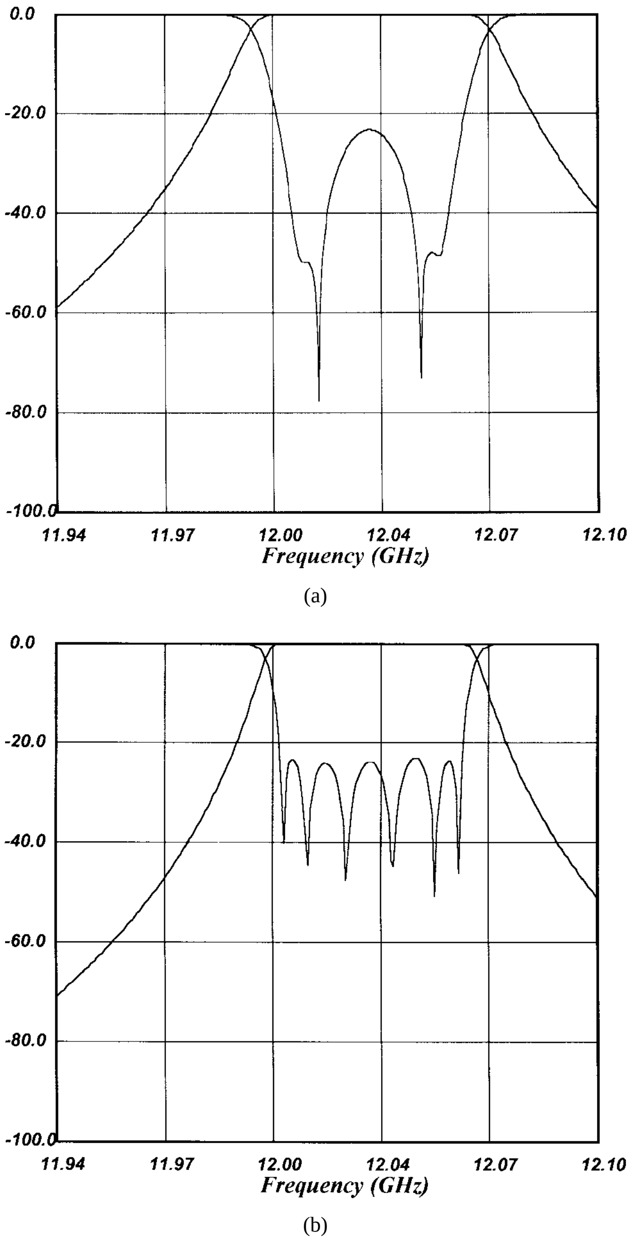


Fig. 1. Cubed second-order Chebyshev CF filter in (a) and a standard Chebyshev of order six in (b).

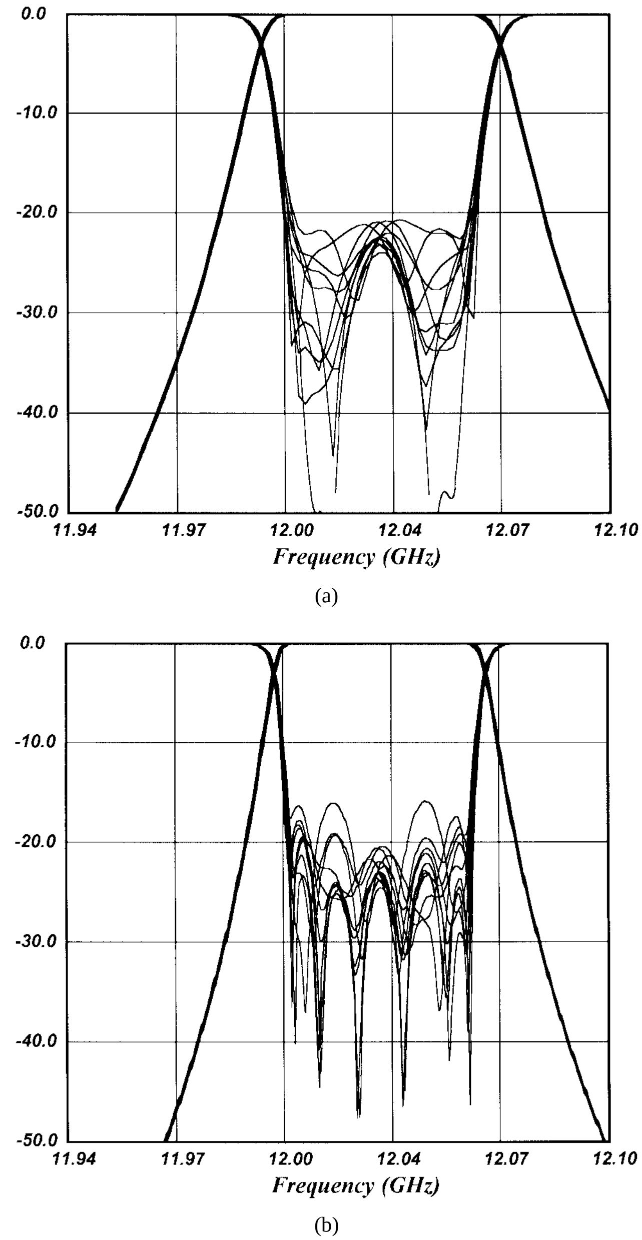
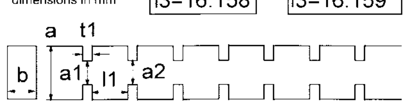


Fig. 2. Effects of random error added to the values of Table I.

TABLE I
DIMENSIONS OF CF AND CHEBYSHEV FILTERS OF ORDER SIX

	Powered	Chebyshev
a= 19.050	t1= 0.646	t1= 1.059
b= 9.525	t2= 0.768	t2= 1.028
a1= 6.425	t3= 1.232	t3= 1.371
a2= 2.950	t4= 1.299	t4= 1.423
a7=a1	l1=15.379	l1=15.421
a3,4,5,6=a2	l2=16.154	l2=16.157
dimensions in mm	l3=16.158	l3=16.159



where Y_e and Y_o are the even- and odd-mode input admittances, respectively.

The transfer, or ABCD matrix of the filter is then formed by

$$\frac{1}{Y_e - Y_o}\begin{bmatrix} Y_e + Y_o & 2 \\ 2Y_eY_o & Y_e + Y_o \end{bmatrix} \tag{9}$$

and the usual ladder or folded array synthesis procedure may proceed [1].

III. EXAMPLES OF CF FILTERS

As examples of filter transfer functions, we limit ourselves to the discussion of CF filters having quasi-Chebyshev characteristics (i.e., the seed functions are Generalized Chebyshev functions). We may select, however, any other function, provided that they are realizable.

Starting, for instance, with a simple higher order Chebyshev function and chaining it with itself to form a Powered

Chebyshev function, we obtain

$$G_m(p) = C_n^m(p) \tag{10}$$

having n zeroes of multiplicity m (i.e., a total of $m \times n$ zeroes). For this case, since all seed functions are Chebyshev, the resulting chained function still has an equiripple behavior between $p = i$ and $p = -i$, allowing exact prescription of the return loss level and bandwidth in the ideal response.

As a further example, we present in Fig. 1(a) a six-poles Powered Chebyshev (cubed second-order Chebyshev) filter with a useful bandwidth from 12.006 to 12.06 MHz. A standard Chebyshev is shown in Fig. 1(b) for comparison. As we can see from Fig. 1(a), the Powered Chebyshev has slightly less selectivity than the pure Chebyshev. However, from the simulation we can see that the CF filter has the same bandwidth as the Chebyshev filter.

The transmission poles of the Powered Chebyshev appear clustered around two frequencies which are the same as for a second-order Chebyshev. It is not possible, however, to clearly distinguish the three coincident poles at the two frequency locations. Ideally, in fact, they should exactly coincide in frequency. In a practical application however, they will appear very near to each other, globally producing a very low value of return loss.

IV. Tolerance to Manufacturing Errors

To evaluate the tolerance to manufacturing errors of CF microwave filters, we have used the two narrow-band filter examples of the previous section. The two filters have been designed using rectangular waveguide resonators coupled by thick inductive steps [2], [3]. Table I gives the dimensions of the filters. To evaluate the effect of manufacturing tolerances we have performed a number of experiments where the dimensions of the filters were corrupted by a random error with a Gaussian distribution. The variance of the distribution was chosen to give a maximum error of about 4 μm. Fig. 2(b) shows the results obtained for seven different experiments using the Chebyshev filter, while Fig. 2(a) shows the same result for the CF filter. As we can clearly see the CF filter is substantially more robust than the Chebyshev filter.

V. Conclusion

A new family of filter transfer functions, called the Chained Function transfer function, has been introduced. The major features of this new filter prototype is that is exhibits a reduced sensitivity to manufacturing error while maintaining a rejection performance that is comparable to the one of a standard Chebyshev filter. The achieved decreased sensitivity has been fully demonstrated using an inductively coupled microwave filter in rectangular waveguide and a full-wave electromagnetic (EM) simulator of proven reliability.

The new family of filter transfer function can be implemented not only in rectangular waveguide technology bat also with any other technology that is currently available. Simple as well as complex transfer functions can be easily implemented thereby providing a very useful new tool for any engineer involved in filter design.

References

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